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GEOMETRICAL INTUITION COMPONENTS FOR DESIGNING MATHEMATICAL TASKS

KOMPONEN INTUISI GEOMETRIS UNTUK MERANCANG TUGAS MATEMATIKA

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Abstract: Geometrical intuition is the ability to visualize, construct, and manage geometrical shapes in the mind when solving geometry problems. Geometrical intuition requires four skills: the ability to construct and manage geometrical figures in mind, perceive geometrical properties, connect pictures to concepts and theories in geometry, and determine where and how to begin when solving geometry problems. This geometric intuition ability is important for developing problem-solving. Therefore, we need a task that can be used to identify and develop students' geometric intuition abilities. This research aims to design a geometric intuition task. We employ design research methods to design geometrical intuition tasks by conducting a literature review on geometric intuition and geometry tasks, creating geometrical intuition tasks, and estimating and noting the possible student responses. This study produced three types of tasks based on the four components of geometric intuition. We provide a list of possible responses that junior high school students may provide, as well as practical suggestions for teachers. We recommend research using our developed task to evaluate students' geometrical intuition.

Keywords: geometrical intuition, geometrical intuition task

Abstrak: *Intuisi geometris adalah kemampuan untuk memvisualisasikan, membangun, dan mengelola bentuk geometris dalam pikiran saat memecahkan masalah geometri. Intuisi geometris membutuhkan empat keterampilan: kemampuan untuk membangun dan mengelola figur geometris dalam pikiran, memahami sifat geometris, menghubungkan gambar dengan konsep dan teori dalam geometri, dan menentukan di mana dan bagaimana memulai saat memecahkan masalah geometri. Kemampuan intuisi geometris ini penting untuk mengembangkan pemecahan masalah. Oleh karena itu, diperlukan suatu tugas yang dapat digunakan untuk mengidentifikasi dan mengembangkan kemampuan intuisi geometri siswa. Penelitian ini bertujuan untuk merancang tugas intuisi geometri siswa. Kami menggunakan metode penelitian desain untuk merancang tugas-tugas intuisi geometris dengan melakukan tinjauan pustaka tentang intuisi geometris dan tugas-tugas geometri, membuat tugas intuisi geometris, dan memperkirakan serta mencatat kemungkinan respons siswa. Penelitian ini menghasilkan tiga jenis tugas berdasarkan empat komponen intuisi geometris. Kami menyediakan daftar kemungkinan tanggapan yang dapat diberikan oleh siswa sekolah menengah pertama, serta saran praktis untuk guru. Kami merekomendasikan penelitian menggunakan tugas yang kami kembangkan untuk mengevaluasi intuisi geometris siswa.*

Kata Kunci*: intuisi geometri, tugas intuisi geometri*

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Geometry is the branch of mathematics concerned with the generalizable properties of shape and space (Nathan et al., 2021). Geometry is also a theory of space consisting of elements. Elements of all geometrical constructs refer to intuition in different and diverse domains of mathematics (Eder, 2021). Some experts argue that geometry requires intuition (Biagioli, 2014; Frege, 1980; Kant, 1998). Intuition is needed in acquiring geometric knowledge and when solving geometric problems.

Intuition is often considered necessary in learning geometry along with logic for connecting to mathematical knowledge (Cifuentes & Gusmão, 2020; Fujita, Jones, & Yamamoto, 2004a; Hersh, 1998). Intuition is defined as a mental (cognitive) process that is unique and contains subjective truth based on procedural memory mechanisms (skills, knowledge, experience, observation, testing or analogy) to find the right strategy for solving problems (Arwanto, Budayasa, & Budiarto, 2018; Fischbein, 1993; Hirza, Kusumah, Darhim, & Zulkardi, 2014; Lin & Chen, 2021; Nosal, 2021). Without intuition, there will be no creativity in mathematics (Wilder, 1967). Intuition can help in several ways, namely building relationships between new experiences and self-perception, simplifying complex math problems, exploring ideas to solve problems and predicting results (Lin & Chen, 2021). With the correct intuition, it can make it easier to solve problems correctly (Babaei, Chaiichi-Mellatshahi, & Najafi, 2012). Although intuition plays an important role, sometimes intuition can hinder problem-solving. As revealed by several experts that if intuition is well developed, the problem-solving process can be improved (Attridge & Inglis, 2015; Babai, Shalev, & Stavy, 2015; Thomas, 2015; Wuryanie, Wibowo, Kurniasih, & Maryam, 2020).

Geometry intuition is one of the core mathematical literacy in China (Zhou & Yang, 2023). Geometry intuition is the mental representation of seeing or reasoning geometrically (Lin & Chen, 2021). In order to train geometric intuition, four skills are required: a) the ability to create and manipulate geometric shapes in an individual's mind; this skill is intended to stimulate the spatial imagination of students. Through these activities, students can develop their intuition by manipulating mental activities with knowledge; b) recognizing geometric properties, this skill aims to develop students' "geometric eyes," particularly in drawing and measuring in relation to definitions, theorems, and riders; c) connect pictures to geometric theorems and concepts; and d) determine where and how to begin solving geometry problems (Fujita et al., 2004a). Each of these characteristics is used in the task design process for this study.

Mathematics education is centered on the design and implementation of assignments, which is used for pedagogical purposes and is crucial in creating a mentally stimulating environment (Geiger, Galbraith, Niss, & Delzoppo, 2021; Hidiroğlu, 2022; Jones & Pepin, 2016; Watson & Ohtani, 2015). Diverse and challenging assignments can be used to present math's challenges in the classroom (Papadopoulos, 2020). A teacher is likened to a designer to make a task that contains student activities, both knowledge and skills, so that the design of the task will produce activities that are used to find concepts, ideas and strategies with various solution paths that can develop mathematical skills (Ainley et al., 2013; Johnson, Coles, & Clarke, 2017; Sinclair, 2003; Watson & Ohtani, 2015; Yerushalmy, Nagari-Haddif, & Olsher, 2017). It is, therefore, essential to provide a literature resource on various classroom assignments that teachers can immediately use. In addition to assignments that can be applied to students or to measure the teacher's ability before the assignment is given to students because maybe the teacher also has particular difficulties in completing student assignments. Like the research conducted by Mutammam et al. (2023) and Mutammam & Wulandari (2023) regarding the ability of teacher and student symbol sense.

The results of the research by Babaei et al. (2012) showed that solving mathematical math problems through intuition will positively affect student performance. Further research results from Eubanks et al. (2010) show that some students who are trained to develop intuition positively influence problem solving skills. From this, intuition can be trained in students by giving math assignments to develop intuition skills.

In terms of geometric intuition, some researchers have studied theoretical intuition (Jones, 1993; Nikulin & Shafarevich, 1994; Shipley, 2015), some others have shown the role of geometric intuition in non-geometric fields (Biagioli, 2014; Chartier, 2002; Durfee & Archibald, 2016), and some researchers discuss geometric intuition in learning (Fujita, Jones, & Yamamoto, 2004b; Fujita et al., 2004a) and provide examples of tasks in developing geometric intuition in class. However, there are no researchers who focus on compiling mathematical tasks that can be used to check and develop junior high school students' geometry intuition by including the intuitive possibilities that arise (both true and false) in completing assignments and suggestions in learning practice (with the inclusion of possible Intuitive answers by students are expected by the teacher to be more anticipatory of errors that may arise in learning geometry). Therefore, the researcher feels the need to conduct research related to that and design some geometric intuition tasks.

Methods

The purpose of this research is to design a geometric intuition task. In conducting this research, we were inspired by research by Jupri & Sispiyati (2021) and Fujita et al. (2004a). We use design research methods to design geometric intuition tasks (Bakker, 2004), particularly in the "a preparation and design phase" with three steps. First, we conducted a literature study on geometric intuition to get some information about geometric intuition. The information we seek includes the definitions and components of geometric intuition, previous research on geometric intuition, geometry problem solving tasks and information on task design, and general and specific mathematical task design on geometric intuition. Second, we designed three types of math tasks corresponding to geometric intuition's four components (Fujita et al., 2004b, 2004a). In designing the geometry intuition task, we drew inspiration from the article by Fujita et al. (2004a), textbooks (Raharjo, 2018), and other sources. Lastly, we estimated and wrote down the possible answers given by junior high school students, both correct and incorrect. We have also added some suggestions for practical needs in class. We present the research flow in Figure 1 to make it easier for readers.

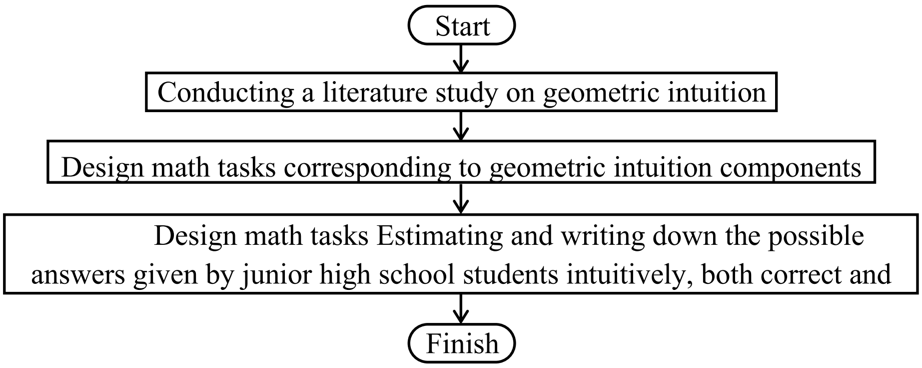


Figure 1. Flowchart of Research Steps

Results and Discussion

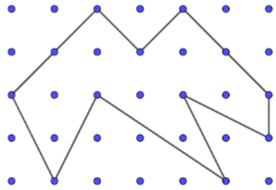
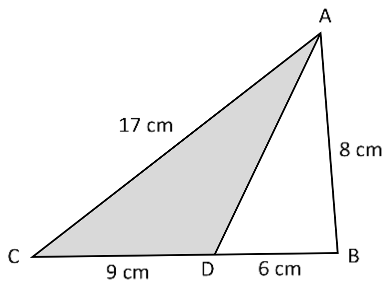
In this section, we present the mathematical tasks we have designed according to the geometric intuition component of Fujita et al. (2004b) and possible responses to answers based on students' intuition in completing the tasks we made. We also add suggestions to teachers for the practical needs of teaching and learning activities in class. Table 1 presents three tasks that can be used to find out or train the four components in students' geometric intuition. Tasks that require geometric intuition in "deciding where and how to start solving problems in geometry" are not specifically crafted. This is because the three tasks can also be seen to know the intuition that arises from students in deciding where and how to complete geometry tasks. Actually, in completing the tasks that we have compiled, the four components of geometric intuition will be used by students. However, a particular geometric intuition component contributes the greatest to task completion. The geometrical intuition tasks we have created are presented in Table 1. Next, we discuss each type of task and the possible responses we can expect.

Our Type 1 task used the geometric intuition component of "creating and manipulating geometric shapes in the mind." This means that in completing this task, it is expected that, intuitively, students can create and manipulate geometric shapes in mind in solving geometric problems. This aligns with a task devised by Treutlien (Fujita et al., 2004a) which focuses on spatial intuition skills. This type of task 1 tends to be more flexible in encouraging students' creativity and imagination because more varied intuitive answers can emerge in students' minds.

In completing this task, we predicted many possible geometric manipulations in the mind that students could perform. One student may only be able to find one variation, while another student may be able to make many variations. This is very dependent on the geometric intuition and creativity that students have. Students with good intuition can choose the one variation that is easiest to solve from many variations of manipulation in their minds. Here are some variations of the results of mind manipulation students might have.

Intuitively students construct polygons into seven triangles, as shown in Figure 2. Using the area of a triangle's formula = $(a \times t)/2$, students find that area Δ I = 2 units area, area Δ II = 2 units area, area Δ III = 1 unit area, area Δ IV = 2 units area, area Δ V = 1 unit area, area Δ VI = 2 units area, and area Δ VII =1 unit so area of the polygon is 12 units area.

Table 1. Geometry Tasks Designed According to the Components of Geometric Intuition

Type of Tasks	Components of Geometric Intuition	Example of tasks
1	Creating and manipulating geometrical figures in the mind	Given a polygon as in the following figure.  The distance between vertically and horizontally adjacent points is 1 unit. What is the area of the polygon above?
2	Perceiving geometrical properties	Given rhombus ABCD with BE perpendicular to AD. If the length of the diagonal BD = 12 cm and AC = 16 cm, determine the length of the segment BE.
3	Relating images to concepts and theorems in geometry	Given triangle ABC. Point D is on the segment BC. AB = 8 cm, AC = 17 cm, BD = 6 cm and CD = 9 cm, as shown in the following figure. Find the area of triangle ADC. 
4	Deciding where and how to start when solving problems in geometry	Assignment Types 1, 2 and 3 can be used to determine these components' geometric intuition.

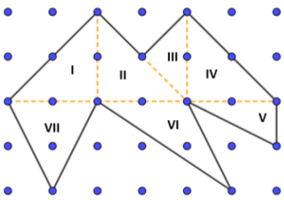


Figure 2. Polygon Partition into Seven Triangles

In order to make our explanation simpler for figures 3 – 8, we will only explain the solution idea without including specific calculations.

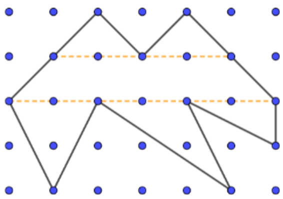


Figure 3. Polygon Partition with Two Line Segments

In Figure 3, students intuitively manipulate polygons by making two horizontal lines and dividing the shape into a trapezoid and a triangle.

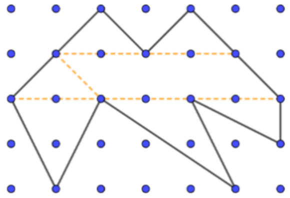


Figure 4. Partition the Polygon into a Parallelogram and Triangles

In Figure 4, students intuitively see parallelograms on polygons, and besides, parallelograms can be made into triangles.

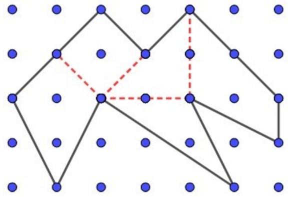


Figure 5. Polygon Partition into Several Triangles and Quadrilaterals

In Figure 5, students intuitively see more variations of triangles and quadrilaterals. These shapes are kites, rhombus/square, right and obtuse triangles and parallelograms.

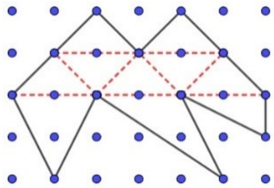


Figure 6. Polygon Partition into Several Congruent Triangles

In Figure 6, students intuitively manipulate shapes into seven congruent triangles and three other triangles. Congruent shapes are easier to calculate because they have the same area.

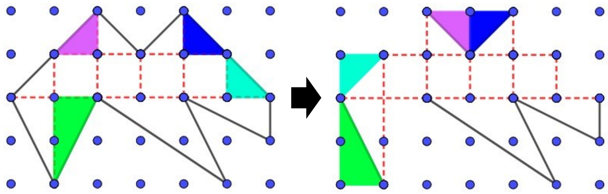


Figure 7. Polygon Manipulation into Several Squares with One Unit Area

In Figure 7, students intuitively focus that 1 square has 1 unit area. So, the polygon is manipulated into several squares with an area of 1 unit. A shape that cannot be manipulated into a unit square must be calculated using the area of a triangle's formula.

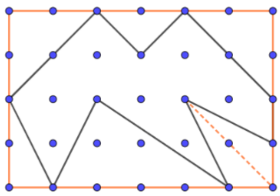


Figure 8. Rectangle Outside the Polygon

In Figure 8, students can intuitively manipulate polygons by adding a rectangle outside the polygon. Then the area of a regular shape can be obtained by calculating the area of the rectangle and subtracting it from the area of the triangles inside the rectangle outside the polygon. If students are able to manipulate geometric shapes in their minds and solve them analytically well, they will get the area of a polygon equal to 12 units area.

An interesting thing from the manipulations above is the area of an obtuse triangle indicated by area IV in Figure 2. Students may experience difficulties if they need more knowledge about the concept of the base and height of a triangle, where the height of the triangle is not only perpendicular to the base but can also be straight riveted with the extension of the base side.

Though geometric intuition by creating and manipulating geometric shapes in mind is the most essential thing in completing this task, more is needed in completing the task. Students must also be able to complete tasks analytically.

Apart from geometric manipulations like in the image above, many other variations exist. In addition, students may not be able to manipulate geometric shapes at all. It is also possible that students are capable of manipulating geometric objects, but some of the figures that their minds have manipulated may not be mainstream, so the area formula is not known. For practical needs in class, the teacher should use this type of assignment to learn or train students' geometric intuition and creativity on triangles and quadrilaterals at the junior high school level. Using this type of task, the teacher can also increase student creativity and discover students' familiarity with geometric shapes and their area formulas.

We designed a Type 2 task using the geometric intuition component "recognizing geometric properties". It means that in completing this task, it is expected that, intuitively, students can recognize and utilize the geometric properties of an object in solving geometric problems. Students are expected to be able to associate the geometric properties of one type of shape with another type of shape. This task has the same purpose as Godfrey (Fujita et al., 2004a) regarding introducing geometric properties. What differs from the assignments prepared by Godfrey is that Type 2 tasks focus on solving problems by recognizing relationships between plane shapes based on their geometric properties. This aligns with knowledge about Van Hiele's geometric thinking abilities (Halat, 2008). Fujita et al. (2004a) presents this task as activities to develop geometric intuition.

In completing this task, we predict that there are at least two correct answers that may appear intuitively in completing the task. The first possibility, intuitively, students recognize, based on geometric properties, that a rhombus is a parallelogram. This is based on Van Hiele's geometric thinking skills required at level 3 (Halat, 2008). Segment BE can be viewed as the height of the parallelogram ABCD with segment AD as the base. Using the Pythagorean formula, AD = 10 cm is obtained. Then students can form the equation "area of rhombus ABCD equals area of parallelogram ABCD" so that (AC×BD)/2 = AD×BE. By solving this equation, we get BE = 9.6 cm.

The second possibility, intuitively, students recognize, based on its geometric properties, that a rhombus is a union of two congruent isosceles triangles. The rhombus ABCD can be constructed from the congruent isosceles triangles ABD and BCD in the above task. So, the area of ΔABD is equal to half of the area of rhombus ABCD. Segment BE can be viewed as the height of ΔABD and segment AD as its base. Using the Pythagorean formula, AD = 10 cm is obtained. Then students can form the equation ½×AD×BE = ½×(½×AC×BD). By solving this equation, we get BE = 9.6 cm.

Students who intuitively cannot recognize the geometric properties of a rhombus will have difficulty solving this problem. Students may also need help solving this problem if the intuition that appears is misleading in solving the problem (Attridge & Inglis, 2015; Babai et al., 2015; Thomas, 2015; Wuryanie et al., 2020). Although geometric intuition by recognizing geometric properties is the essential thing in solving this task, it is still not enough. Students must be able to apply geometric properties and then solve them analytically. For practical needs in class, teachers can use this assignment to find out or train students' geometric intuition on triangles and quadrilaterals or the Pythagorean theorem at the junior high school level. This type of task can also improve the "geometry eye" ability to view a geometric shape.

We designed Type 3 tasks according to the geometric intuition component of "connecting images with concepts and theorems in geometry". It means that in completing this task, it is expected that, intuitively, students can connect the image representations on the assignment with concepts and theorems in geometry or vice versa. In the question Type 3 above students must connect the picture with the Pythagorean theorem. In contrast to the task made by Fujita et al. (2004a), this type of task focuses on intuition in solving problems rather than developing intuitive geometric concepts using a geoboard.

In solving Type 3 tasks, we predict that with their geometric intuition, students will realize that ΔBCD is a right triangle based on the picture and the combination of the three side lengths, a combination of Pythagorean triples. With an understanding of the concept of the base and height of a triangle, students view side BC as the height of ΔABC so that if drawn correctly, it will look like Figure 9.

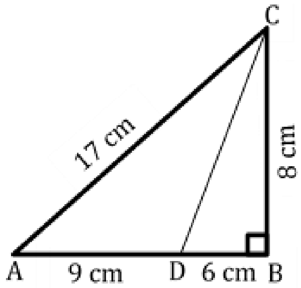


Figure 9. Exact Figure Representation in Task 3

Based on Figure 9, then the area Δ ADC = ½×AD×BC = 36 cm^2

Another possibility, students intuitively realize that ΔABC is a right triangle. To check the truth of this intuitive knowledge, students analytically check the truth using the inverse of the Pythagorean theorem: $8^2+15^2 \ldots 17^2 \leftrightarrow 64+225 \ldots 289 \leftrightarrow 289 = 289$. Furthermore, with their intuition, students see that ΔABC is formed from two triangles, ΔACD and ΔBCD. So, finding the area of ΔACD can be done by determining the difference between the areas of ΔABC and ΔCBD. Furthermore, using analytical skills, students perform calculations to obtain an area of ΔACD = 36 cm^2.

Another possibility, students intuitively assume that segment AD is the height of the triangle. This aligns with Dvora & Dreyfus (2004) that assumptions based on diagrams may occur in solving geometry tasks. Assuming that ΔBCD is a right triangle because the side lengths are 6 cm and 8 cm, which are part of the Pythagorean triple combination 6, 8, 10. Using analytical skills, students find the length of the segment CD. CD = $\sqrt{(6^2+8^2)} = \sqrt{100} = 10$. With their intuition, students perceive that ΔABC is a right triangle as a result of ΔBCD with a right angle at B. With their intuitive understanding of the wrong concept, students view the side CD as the height of ΔACD so that the area of ΔACD = ½×AD×CD is obtained. = ½×9×10 = 45 cm^2. This is a wrong answer because intuition tends to be based on assumptions and a lack of understanding of the concepts of base and height.

We deliberately made the pictures in task Type 3 so that ΔABC is not easily seen as a right triangle. It is hoped that with its contents, students will realize that ΔABC is a right triangle based on the combination of side lengths ΔABC, which is a Pythagorean triple. If deemed necessary, maybe they will prove the truth. Students who cannot connect pictures with the concepts and the Pythagorean theorem, so they realize that ΔABC is a right triangle, will have difficulty solving this problem. Wrong intuition, such as assuming that the CD side is the height of a triangle, will make it difficult for students to complete this task. Teachers can use this type of assignment for practical needs in class to find out or train students' geometric intuition on the Pythagorean theorem material at the junior high school level. This task can also be a means for teachers to evaluate and justify concepts that students do not yet understand. The teacher can also know that wrong intuitions hinder solving geometric problems.

Based on the results and discussion of this problem, the teacher can be more anticipatory of the possibility of good intuitive answers from students and how to direct them. Besides that, the teacher can also know that wrong intuitions hinder solving geometric problems. Therefore, the teacher can be more sensitive and anticipate possible errors from the start.

Conclusions and Suggestions

Based on the explanation in the previous section, we draw several conclusions. Based on the components of geometric intuition, we designed three types of math tasks. We have provided predictions about the intuitive responses that may arise for each type of task. We have added some suggestions about using assignments in classroom math practice. We recommend, for further research, using the task we have designed to assess the geometric intuition of students at the junior high school level. Each task can be used independently or all three can be used together depending on the needs of the researcher. We've also added some suggestions for practical needs in class.

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